A-level
FURTHER MATHEMATICS
7367/3D

Paper 3 Discrete

Mark scheme

June 2019

Version: 1.0 Final

1 9 6 A 7 3 6 7 / 3 D / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

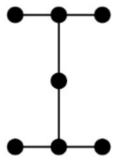
Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

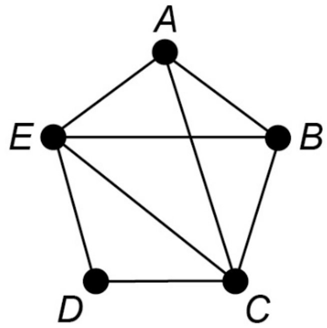
AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking Instructions	AO	Marks	Typical Solution
1	Selects correct answer	AO1.1b	B1	**A**
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
2	Selects correct answer	AO1.1b	B1	
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3(a)	Uses the optimal objective row to find at least one correct inequality in k (Condone strict inequality or inequalities)	AO3.1a	M1	$k^2 + k - 6 \geq 0$ $k - 1 \geq 0$ $k = 2, -3$ $k \geq 2$, $k \leq -3$ satisfy the quadratic inequality, but we also require that $k \geq 1$ to satisfy the linear inequality. Hence, the range of values for k is $k \geq 2$
	Calculates both correct critical values of the quadratic inequality (Allow equals signs and inequalities)	AO1.1b	A1	
	Uses the requirement that both inequalities must be satisfied simultaneously to deduce the correct range of values k must satisfy (Do not condone strict inequality)	AO2.2a	A1	
(b)	Writes down correctly the value of the variable s	AO1.1b	B1	$s = 0$
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
4(a)	Draws a correct labelled graph with edges and vertices.	AO1.1b	B1	
(b)	Recalls Euler's formula correctly.	AO1.2	M1	$v - e + f = 2$
	Identifies that the graph *P* has 5 vertices and 8 edges and uses Euler's formula to show f = 5.	AO3.2a	A1	5 vertices $\Rightarrow v = 5$ 8 edges $\Rightarrow e = 8$ $f = 2 + e - v$ $f = 2 + 8 - 5$ Therefore $f = 5$ and so the graph *P* has exactly 5 faces.
(c)	Identifies that vertices *A* & *D* are not adjacent and *B* & *D* are not adjacent (PI)	AO3.1a	M1	Vertices *A* & *D* are not adjacent. Vertices *B* & *D* are not adjacent. degree of *A* = 3 degree of *B* = 3 degree of *D* = 2
	Uses the correct degree for vertices *A*, *B* and *D* (PI) and finds the correct sums of degrees for *A* & *D* and *B* & *D* (Condone calculations for *A* & *B*)	AO1.1b	A1	*A* & *D*: 3 + 2 = 5 *B* & *D*: 3 + 2 = 5
	Constructs rigorous mathematical argument by explaining that the condition of Ore's theorem holds true for both pairs of non-adjacent vertices with n = 5, which proves that *P* is Hamiltonian.	AO2.1	R1	We have shown that for each pair of vertices *X* and *Y* of *P* which are not adjacent, degree of *X* + degree of *Y* ≥ 5 As *P* has 5 vertices, by Ore's theorem, the graph *P* is Hamiltonian.
	Total		**6**	

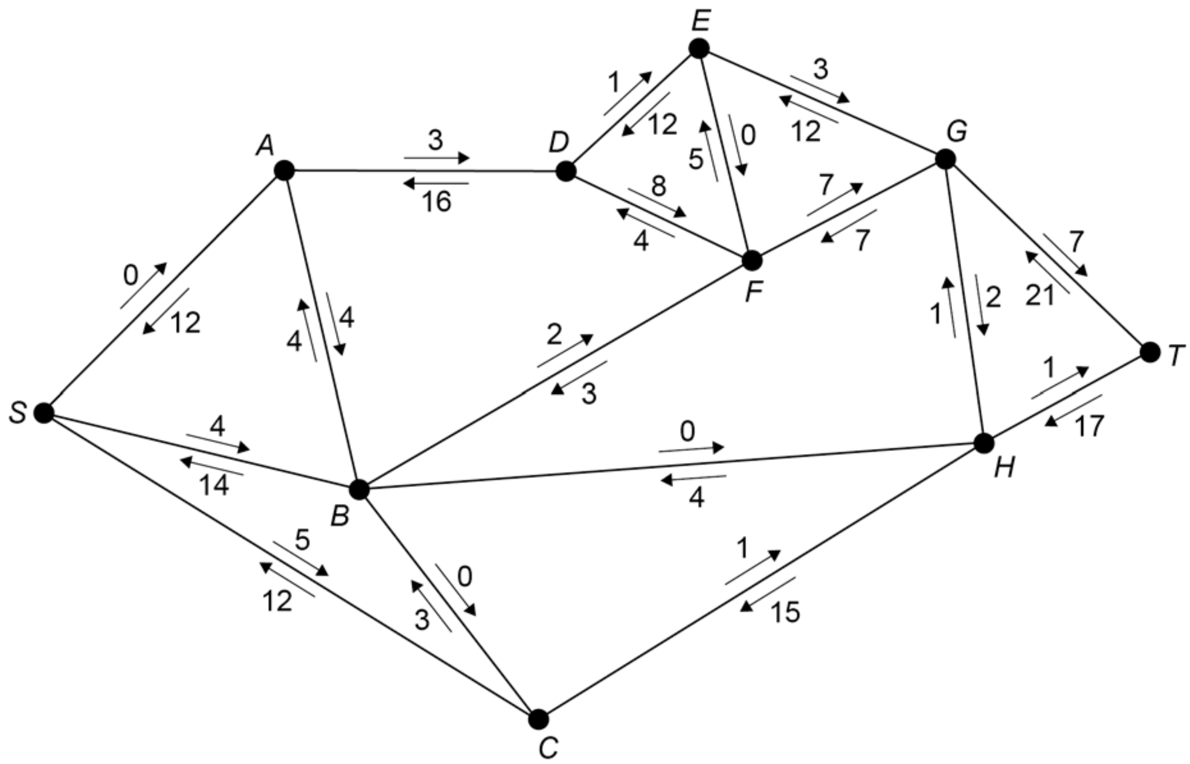
Q	Marking Instructions	AO	Marks	Typical Solution
5(a)(i)	Finds correctly $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^n$ for n = 2, 3 or 4	AO1.1a	M1	$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = C$ $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^3 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = D$ $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A$
	Finds correctly $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^n$ for n = 2, 3 and 4	AO1.1b	A1	
	Concludes that repeated multiplication of $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ by itself results in producing every element of *G*, which proves that $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is a generator of *G*.	AO2.1	R1	As each of $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^2$, $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^3$ & $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^4$ is a different element of *G*, then we have proved that $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is a generator of *G.*

5(a)(ii)	States at least one correct mapping between an element of *G* and an element of *H* (Condone poor notation) or States that *G* is a cyclic group of order 4.	AO3.1a	B1	$B=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\mapsto \mathrm{i}$ $B^2=C=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}\mapsto -1=\mathrm{i}^2$ $B^3=D=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\mapsto -\mathrm{i}=\mathrm{i}^3$ $B^4=A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\mapsto 1=\mathrm{i}^4$ Both groups have the same structure which is preserved by the one-to-one mapping identified above, and so *G* is isomorphic to *H*.
	Uses correct mathematical notation/language to show each correct mapping between the elements of *G* and the elements of *H* or Identifies i as a generator of *H*.	AO2.5	B1	
	Deduces and states that there is a **one-to-one** mapping between each element of *G* and an element of *H* or Deduces that *H* is a cyclic group of order 4.	AO2.2a	E1	
	Completes rigorous argument to conclude that *G* is isomorphic to *H*.	AO2.1	R1	

5(b)(i)	Identifies the order of *H*.	AO1.1b	B1	The order of *H* is 4. 4 is not divisible by 3, and so *H* cannot have a subgroup of order 3 by Lagrange's theorem.
	Explains that the order of *H* is not divisible by 3, and so by Lagrange's theorem *H* cannot have a subgroup of order 3.	AO2.4	E1	
(b)(ii)	Finds one subgroup of *H*. (Condone poor notation such as circular brackets).	AO1.1a	M1	$\{1,\ -1\}$
	Finds a second subgroup of *H*. (Condone poor notation such as circular brackets).	AO1.1b	A1	$\{1\}$
	Finds all three subgroups of *H* and no others, which include *H* itself and the trivial subgroup, using the correct notation.	AO2.5	A1	$H = \{1,\ -1,\ \mathrm{i},\ -\mathrm{i}\}$
	Total		**12**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Sets up a model by identifying the problem as a route inspection problem and noting that *B*, *C*, *F* and *G* are odd-degree nodes (PI)	AO3.3	M1	Odd nodes: *B*, *C*, *F*, *G* Shortest Distances *B*–*C*: 160 *B*–*F*: 60 *B*–*G*: 235 *C*–*F*: 220 *C*–*G*: 75 *F*–*G*: 295 Pairings (*B*–*C*)(*F*–*G*) = 455 (*B*–*F*)(*C*–*G*) = 135* (*B*–*G*)(*C*–*F*) = 455 Distance of all streets in town centre = 1090 m Minimum distance the traffic warden can travel whilst monitoring is 1090 + 135 = 1225 m Hence, the short possible time is $\frac{1225}{4800} \times 60 = 15$ minutes
	Uses the model to find all three correct totals for the pairs of shortest distances.	AO3.4	A1	
	Determines correctly the total length of all of the streets in the town centre.	AO1.1b	B1	
	Determines correctly their minimum total distance that the traffic warden will cover during the journey.	AO1.1a	M1	
	Determines the correct least possible time, to the nearest minute, from their minimum total distance. (CAO but condone lack of units)	AO3.2a	A1	
(b)	Gives a plausible reason as to why the actual time taken by the traffic warden may be different, and explains the consequences of the reason.	AO3.5b	E1	The traffic warden may need to stop to issue a fine to vehicles which will reduce their average speed, causing an increase in the time taken to complete the walk around the town centre's streets.
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(a)(i)	Calculates correctly the value of the cut (Condone no units).	AO1.1b	B1	44 litres per second
(a)(ii)	Explains correctly the meaning of their value of the cut in the context of the question, including units and pipes.	AO3.2a	B1F	The maximum flow of water through the system of water pipes is less than or equal to 44 litres per second.
(b)(i)	Finds a correct potential increase and decrease for the arcs *SA*, *SB*, *SC*, *GT* and *HT.* (Condone numbers reversed)	AO1.1a	M1	
	Determines correctly the potential increase and decrease for each arc.	AO1.1b	A1	

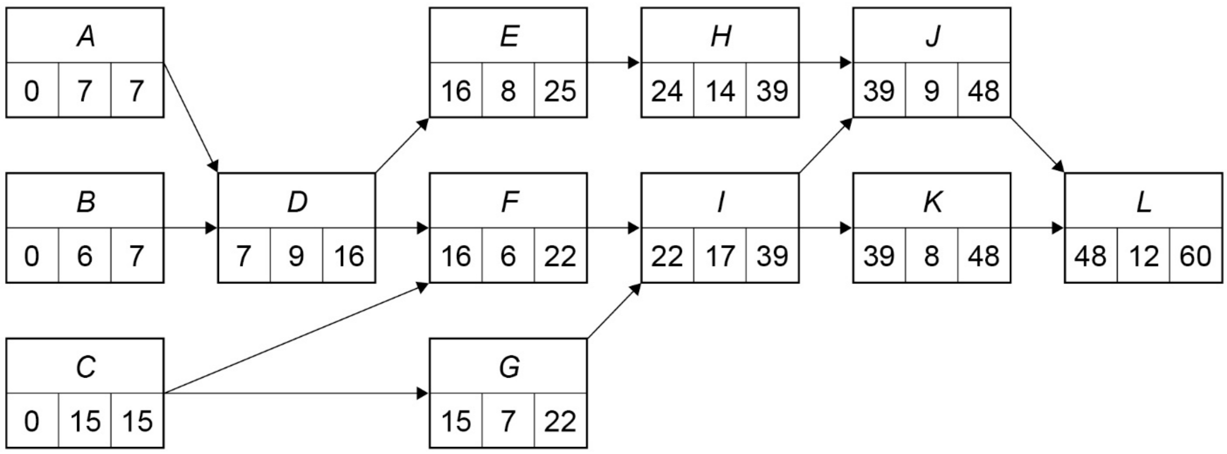


<table>
<tr><td rowspan="4">(b)(ii)</td><td>Finds correctly one augmenting path and flow.</td><td>AO1.1a</td><td>M1</td><td rowspan="4"><table><tr><th>Augmenting Path</th><th>Flow</th></tr><tr><td>SCHT</td><td>1</td></tr><tr><td>SBADFGT</td><td>3</td></tr><tr><td>SBFGT</td><td>1</td></tr><tr><td>SCBFGT</td><td>1</td></tr></table>
Maximum flow = 38 + 6
= 44 litres per second.</td></tr>
<tr><td>Finds correctly two augmenting paths and the flows.</td><td>AO1.1b</td><td>A1</td></tr>
<tr><td>Finds correctly at least three augmenting paths and the flows.

The sum of the augmenting flows must be 6.</td><td>AO1.1b</td><td>A1</td></tr>
<tr><td>States the maximum flow.
(Condone no units).</td><td>AO3.2a</td><td>B1</td></tr>
<tr><td rowspan="2">7(c)</td><td>Explains correctly that pipe BF should be upgraded.</td><td>AO2.4</td><td>E1</td><td rowspan="2">Pipe BF should be upgraded as this will result in an increase of 2 litres per second through the system of pipes.

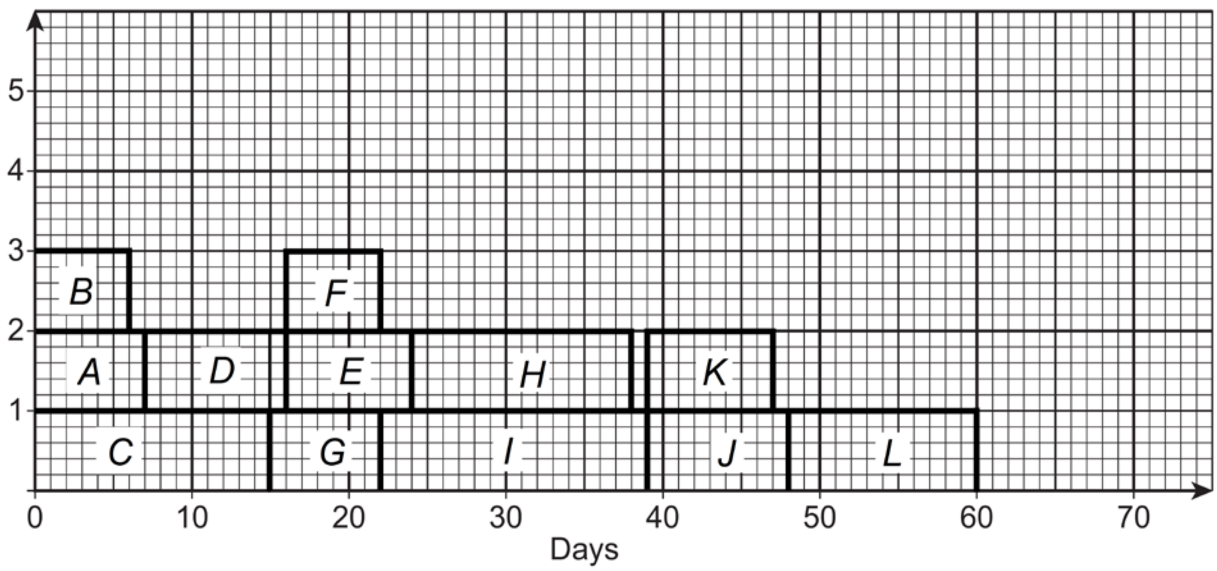
This will increase the maximum flow through the network to 46 litres per second.</td></tr>
<tr><td>Deduces their new maximum flow through the network is 2 more than their previous maximum flow.
(Condone no units)</td><td>AO2.2a</td><td>B1F</td></tr>
<tr><td></td><td>Total</td><td></td><td>10</td><td></td></tr>
</table>

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)(i)	Constructs an activity network with at least 10 labelled activities drawn and at least 4 connections.	AO3.1a	M1	
	Activity network fully correct with all activities and connections. (Condone omission of arrows)	AO1.1b	A1	
(a)(ii)	Finds correctly the earliest start time for activities *E*, *F* and *G.*	AO1.1a	M1	
	Finds correctly the earliest start time for activities each activity on the network.	AO1.1b	A1	
	Finds correctly the latest finish time for each activity on the network.	AO1.1b	B1	



Q	Marking Instructions	AO	Marks	Typical Solution
(b)	Correctly identifies both critical paths and no others.	AO3.2a	B1	*A–D–F–I–J–L* and *C–G–I–J–L*

8(c)(i)	Draws a resource histogram with a consistent vertical scale with at least 10 labelled regions. Must have a correct critical path on the bottom row. (Condone floats, but do not condone overhangs).	AO3.1b	M1	
	Draws the correct resource histogram.	AO1.1b	A1	



(c)(ii)	Evaluates the effect of only having two members of the team working on the project by noting that activities *A* and *B* cannot be carried out simultaneously. (PI by Figure 5 or by increase of 6 days)	AO3.1b	M1	*A* and *B* cannot be done at the same time as this would require 3 members of the team. Therefore, *B* must follow *A* which increases the earliest start time for *D* to 13. This leads to a new minimum completion time of 66 days, as activities *E*, *F* and *G* will only take two team members to complete.
	Explains correctly that the minimum completion time for the project increases to 66 days.	AO2.4	R1	
	Total		**10**	